Science And Technology – Magic Squares II

Introductory Note: The **Magic Squares** series consists of two issues with 9 stamps. The first issue was published on 9 October 2014. In this issue the last 3 stamps with the facial values of 8, 1 and 6 patacas are published.

The theme of *Magic Squares* is transversal in Chinese and Western cultures, **Macao Post** intends to publicize and promote the **scientific and cultural aspects** of this theme as well as to create a **unique product** in the history of Philately.

Due to limited space in the **Information Brochures**, additional explanations of the **Souvenir Sheets**, **Sheetlets**, **First Day Covers** and **Stamps**, as well as some technical terms used to characterize and define *Magic Squares* can be found at the following **Site** of *Macao Post*:

- **Magic Squares Issues I** and **II –** http://goo.gl/lOHEqo.

Souvenir Sheet: Method of Knight's Tour

There are several general methodologies to construct *Magic Squares* depending on the **Class** and **Order**. Among them, the following can be mentioned: **La Loubère** or **Siamese**, **Bachet de Méziriac**, **Philippe de la Hire**, **John Lee Fults**, **Ralph Strachey**, **Knight's Tour**, **Dürer**, etc.

In the **Souvenir Sheet** of this issue, the **Method of Knight's Tour** is used to construct a **Magic Square of Order 16** with a **Closed** or **Reentrant Tour**.

This method consists, starting in an **Initial Cell**, in which the number 1 is allocated, to fill the *Cells* numerically and sequentially from 1 to n^2, of a **Square of Order n**, using the characteristic movements of a *Knight Jump*, as in the Chess game.

Once the **Tour** is established, between the **Initial Starting Cell** and the **Final Arriving Cell**, if it is possible to jump from the *Final Arriving Cell* to the *Initial Starting Cell* with a legal *Knight movement*, the *Tour* is called **Closed** or **Reentrant** and, in this case, the *Initial Starting Cell* can be anyone. On the contrary, the *Tour* is called **Open** or **Non-reentrant**.

The interest aroused by the creation of *Magic Squares* using the *Method of Knight's Tour* in different dimension *Boards*, led to studies that concluded not to be possible to exist a *Magic Square Tour* in $n \times n$ *Boards* with *n Odd*. However, it is possible for *Boards* of **Order 4 k×4 k**, with **k>2**.

The *Magic Square of Order 16* with a *Closed Tour* in the *Souvenir Sheet* of this issue was published by the author **Joseph S. Madachy**, in 1979.

As shown in the *Souvenir Sheet*, the allocation of the numbers 1 to 256 in the *Cells* is sequential and complies with the *Knight Jump* rule of Chess game. The **Magic Sum** is 2056.

Sheetlet

The **Sheetlet** presents a disposition for the face values of the stamps (1 to 9 patacas) equal to the disposition that the numbers 1 to 9 occupy in the **Luo Shu Magic Square**.

In this issue, the last three stamps with the **face values** of **8**, **1** and **6** patacas, corresponding to the **Inferior Row** as mentioned in the *Introductory Note* are issued.

On the **Lateral Margins**, two *Magic Squares* **Tiling Schemes** are presented, proposed by **David Harper** which correspond to the **binary** and **decimal numerical bases**.

First Day Cover: Yang Hui Magic Circles

The XIII Century was probably one of the most important periods in the History of Chinese Mathematics, with the publication of **Shu Shu Jiu Zhang** (**數書九章**), 1247, by **Qin Jiu Shao** (**秦九韶**) and **Ce Yuan Hai Jing** (**测圓海鏡**), by **Li Ye** (**李冶**), followed 15 years later, by the works of **Yang Hui** (**楊輝**).

Yang Hui (1238-1298), a Chinese mathematician born in Qiantang (錢塘) (modern Hangzhou (杭州)), Zhejiang Province (浙江省), during the late Song Dynasty (宋朝) (960-1279). His best known work was **Yang Hui Suanfa (楊輝算法)**, **Yang Hui's Methods of Computation,** which was composed of 7 volumes and published in 1378.

The topics covered by *Yang Hui* include Multiplication, Division, Root-extraction, Quadratic and System Equations, Series, Computations of Areas of Polygons as well as *Magic Squares*, *Magic Circles*, the **Binomial Theorem** and, the best known work, his contribution to the **Yang Hui's Triangle**, which was later rediscovered by **Blaise Pascal**, 1653.

The Bottom Left Corner of the *First Day Cover* presents the **Yang Hui Magic Circles**. These **Nine Circles** are composed by **72 Numbers**, from 1 to 72, with each individual *Circle* having **8 Numbers**. The **Neighboring Numbers** make **Four Additional Circles**, each with *8 Numbers*, the total **Sum** *of the 72 Numbers* is **2628** and the **Sum** *of the 8 Numbers* **in each Circle** is **292**.

Stamp (3/3) : Inder Taneja – IXOHOXI 88

Inder Taneja, Professor of the Department of Mathematics of University of Santa Catarina, Brazil, from 1978 to 2012. He has published more than 100 research papers in internationally renowned journals.

IXOHOXI Magic Squares are a special series that not only show common properties like other *Magic Squares*, as well as being **Pandiagonals**, but also include alternative properties such as **Symmetries**, **Rotations** and **Reflections**.

The word *IXOHOXI* is itself a **Palindrome** and *Symmetric* (*Reflection*), in relation to its center "H". As the 10 digits (0 to 9) use the number style of a **7 Segments LED Display**, in which only 5 digits (0, 1, 2, 5 and 8) remain the same after a **180 Degrees Rotation**. It should be noted that the 4 digits (**0**, **1**, **2** and **5**) used to construct the **Magic Square of Order 4**, are precisely the same digits that constituted the year **2015**, year of its publication as a stamp.

Taking into consideration the 5 digits and their *Symmetric Properties*, *Inder Taneja* created the *IXOHOXI Universal 88 Magic Square*, reproduced in this stamp, has the following properties:

The *Magic Square* still remains a *Magic Square*:

- After a **Rotation of 180 Degrees**;
- After **changing the order of the digits** in the *Cell* numbers, i.e. 82 to 28;
- If it is **seen in a mirror**, or **reflected in water** or **seen from the back** of the sheet;
- The *Magic Sum S* of the *Magic Square of Order 4* is equal to **88**, number that also enjoys *Symmetrical* properties.

Stamp (3/1): McClintock / Ollerenshaw – Most Perfect

A **Most-Perfect Magic Square** is a **Pandiagonal Magic Square of Doubly Even Order** – with additional two proprieties:

- The *Cells* of any square of **Order 2**, (**2×2 Cells**) extracted from it, including **Wrap-Around**, sum up to the same constant value, $\mathbf{2(1+n^2)}$;

- Along the **Main** or **Broken Diagonals**, any two numbers separated by **n/2 Cells**, are a **Complementary Pair**, i.e. sum $\mathbf{1+n^2}$.

In the case of the *Most-Perfect Magic Square of Order 8* reproduced in the stamp, the mentioned properties show the following results:

- $2(1+n^2) = 2(1+8^2) = 130$ E.g.: (59+38+7+26) = (48+33+18+31) = 130
- $(1+n^2) = (1+8^2) = 65$ E.g.: (1+64)=(34+31) = (25+40) = 65

All the *Pandiagonal Squares of Order 4* are *Most-Perfect*. However, when *n>4* the proportion *Pandiagnal to Most-Perfect* decreases as *n* increases.

It is not possible to establish the history of *Most-Perfect Magic Squares* without to mention **Kathleen Timpson Ollerenshaw**. In 1982, with **Hermann Bondi**, she developed a **mathematical analytical construction** that could verify the number **880** for the **essentially different** *Magic Squares of Order 4*. After this achievement she began to study *Pandiagonal Magic Squares* based on works published by **Emory McClintock** in 1897. After several years, in 1986, *Kathleen Ollerenshaw* published a paper where, making use of *Symmetries*, she proved that there are **368640** *essential different Most-Perfect Magic Squares of Order 8*.

Step by step, finally she could discover how to construct and how to count the total number of *Most-Perfect Magic Squares* of all with an **Order Multiple of 4**.

Together with **David Brée**, who helps her organize her research notes and proof-reading, they finally published the book “**Most-Perfect Pandiagonal Magic Squares: Their Construction and Enumeration**” in 1998.

Stamp (3/2): David Collison – Patchwork

David M. Collison (1937-1991) was born in United Kingdom and lived in Anaheim, California. He was a fruitful creator of *Magic Squares* and **Cubes**. He specialized in **Generalized Shapes** from which he created the **Patchwork Magic Squares**.

A *Patchwork Magic Square* is an **Inlaid Magic Square –** one *Magic Square* that contains within it other *Magic Squares* or **Odd Magic Shapes**. The most common *Shape* is **Magic Rectangle**, but **Diamond**, **Cross**, **Elbow** and **L** *Shapes* can also be found.

These *Shapes* are *Magic* if the *Sum* in each **Direction** is proportional to the number of *Cells*. The **Patchwork Magic Square of Order 14** reproduced in this stamp has the following proprieties:

- Contain: Four **Order 4 Magic Squares**, *4×4*, in the **Quadrants**; One **Magic Cross**, *6×6*, in the **Centre**; Four **Magic Tees**, *6×4*, on the **Centre Sides**; Four **Magic Elbows**, *4×4*, in the **Corners**.
- All the *Shapes* sum to a **Constant** which is directly proportional to the number of *Cells* in a *Row*, *Column* or *Diagonal*: $\mathbf{S_2=197}$; $\mathbf{S_4=394}$; $\mathbf{S_6=591}$; $\mathbf{S_{14}=1379}$.

Bibliography: Available at the website of *Macao Post* mentioned above.

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